1. (a) Consider the smallest 4-digit number that is the sum of three consecutive natural numbers. What is the smallest of these three consecutive natural numbers?

   (b) Half of a number x is Z less than twice the square of the sum of the first five prime numbers, where $Z = \frac{a}{9}$. What is x?

   (c) If the average of five consecutive integers is Z, where $Z = \frac{b-32}{30} - 100$, what is the largest of these five consecutive integers?

   (d) Let $Z = 3c$. A student takes a 50 question test, where Z points are awarded for each correct answer and 6 points are deducted for each wrong answer. If the student answers every question and scores a 30, how many questions did that student answer correctly?

2. (a) You roll two dice, one colored red and one colored gold. What is the probability you roll a sum of 8.

   (b) Let $Z = 36a$.

   On an exam, Bob is told to subtract Z from a given number and then divide by 9. Instead he subtracted 9 and divided by Z to get 46. If the problem was worked correctly, what would the answer be?

   (c) Let $Z = b - 17$.

   Solve the following system of equations for c.

$$2x - c = Z$$
$$3x + 4c = -14.$$

   (d) Let $Z = c + 8$

   Suppose a set $A = \{1, 2, 3, \ldots, 12\}$. How many subsets of size Z are there?

3. (a) What is the largest 3-digit integer whose square root is prime?

   (b) Let $Z = \sqrt{a} - 10$. A rectangular box has length Z cm, width 12 cm, and height 16 cm. Find the length of the longest line segment that can be contained in this box.

   (c) Let $Z = b - 23$. Starting with a number x, form a second number that is Z more than x, a third number that is $2Z$ more than x, and continuing this pattern until we have a sixth number which is $5Z$ more than x. If the sum of these six numbers is the least common multiple of 117 and 72, what is x?

   (d) Let $Z = c + 87$. A book was printed that uses a total of Z digits for the page numbers, where $Z > 190$. If the first page is numbered 1, how many numbered pages are in the book?

4. (a) Consider the following true statements:

   - Each heist is made by three minions.
   - Each minion makes exactly two heists.
   - Every pair of minions make at most one common heist.
   - There is at least one heist.

   What is the minimal number of possible heists?

   (b) Let $Z = a + 12$.

   Judy Hopps has been put on parking duty in Zootopia. Judy is determined to give more than 200 tickets, in fact she gives 215 tickets. There are Z officers on parking duty and an average of 80 tickets have been given. If we remove Judy, what is the average number of tickets for all other officers on parking duty?

   (c) Let $Z = \dfrac{b+1}{12}$

   Hiro needs to reprogram Baymax. In Baymax's chip, a word is translated into a sequence of 0's and 1's. How many sequences of length Z are there if there must be at most two 0's.

   (d) Let $Z = c - 17$.

   Anna buys a bunch of carrots for Sven and the other reindeers. Sven eats five times as many as one reindeer. The remaining Z reindeer eat 3 each. What is the smallest number of carrots possible in the bunch that Anna bought?

5. (a) In how many distinct ways can all of the letters in $BUMBLEBEE$ be arranged so that M and L are adjacent?

   (b) Let $Z = a - 2236.5$. Suppose that John leaves his house at 7:00 and walks 2.5 miles to school at a rate of 3 miles per hour. If his sister Kayla leaves 5 minutes after he did and walks the same route as John at Z miles per hour, how many minutes will it take Kayla to catch up with John?

   (c) Let $Z = \frac{b}{7}$. It's 1 AM and you can't sleep, so you decide to calculate the amount of money that you would earn in the next 24 hours if you made $\$Z$ every time that the hands of a non-digital clock were 90^o apart. How much money would you make (in dollars)?

   (d) Let $Z = \frac{c}{10} - 13$. If there are Z students who want to begin a club, in how many ways can four different people be chosen for president, vice president, secretary, and a treasurer if three of the members refuse to serve in these offices?

6. (a) Barney, Robin, Ted, Lily and Marshall are racing through the streets of NYC to get to a restaurant where Woody Allen has been spotted. In how many possible orders can they arrive at the restaurant?

   (b) Let $Z = \dfrac{a}{30}$

   Marshall is waiting for his results on the Bar Exam. His password to receive his results is a string of letters followed by a string of the integers 0-9. If there are no restrictions on the numbers, how many ways can he create a string of Z numbers?

   (c) Let $Z = \sqrt{b}$

   Lily is working as an art consultant for the Captain. She finds an elephant painting that she recommends for him to buy. He buys the painting for $\$100,000$. After Z years, the painting is worth $\$4,000,000$. If the interest on the painting used simple annual interest, what was the interest rate (as a percent)?

   (d) Let $Z = c + 37$.

   Ted has always wanted to build part of the NYC skyline. He finally has the chance to build the new GNB building, and wonders how tall it should be. He selects 5 other building to compare his to. He knows that the mode of those 5 buildings is 60, the median is 15 more than the mode, and average is Z. He also knows the tallest is the One World Trade Center at 104-stories. How many stories is the 5^{th} building?

7. (a) Suppose that there are three houses in a row and two children in each house: in one house lives two girls, in another lives two boys, and in the third is a boy and a girl. If you randomly walk into one of these houses and see a boy, what is the probability that the other child in the house is also a boy?

   (b) Let $Z = 18a$. Alex has a pair of dice, but instead of the usual labeling, for each die each side has exactly one of the numbers: 2, 3, 5, 7, 12, and 15. If Alex rolls this pair of dice, what is the probability of getting two numbers whose sum is less than Z?

   (c) Let $Z = 10(18b - \frac{3}{2})$. Amber's age is currently twice what Joe's age was five years ago. In three years the sum of Joe's and Amber's ages will be Z. How old is Amber today?

   (d) Let $Z = \frac{c}{2} - 10$. A survey was conducted to determine what ice cream flavor customers like, given the choice of vanilla or chocolate. Of those surveyed, $\frac{1}{Z}$ didn't like chocolate, $\frac{1}{5}$ did not like either flavor, $\frac{2}{7}$ didn't like vanilla, and 183 liked both chocolate and vanilla. How many people were surveyed?

8. (a) Consider an isosceles triangle with side lengths 26 and base length of 48. Find the area of the triangle.

   (b) Let $Z = \dfrac{a}{60}$.

   You are making a part iced tea and part lemonade drink. You take two glasses. In the first glass, you pour Z ounces of iced tea. In the second glass you pour Z ounces of lemonade. You pour $1/2$ of the first glass into the second. You mix the second glass thoroughly and pour $1/2$ back into the first glass. What fraction of the liquid in the first glass is lemonade?

   (c) Let $Z = 5b + 6$.

   Compute the sum of all of the roots of $(3x + 4)(x - 2) + (3x + 4)(x - Z) = 0$.

   (d) Let $Z = 6c$.

   Let Z be the sum of two numbers. If we add 4 to each number and then double the result of each number, then what is the sum of the final two numbers?