1. Relay 1:

   (a) A man born in the first half of the nineteenth century was x years old in the year x^2. In what year was he born?

Solution: 1806

   (b) Let $Z = \dfrac{a-6}{90}$.

   The points A, B, and C are on a circle O. The tangent line at A and the secant BC intersect at P, where B lies between C and P. If the line segment from B to C has length Z and the line segment from P to A has length $10\sqrt{3}$, find the length of the line segment from P to B.

Solution: 10

   (c) Let $Z = b - 4$.

   Find the sum of the numerical coefficients in the expansion of the binomial $(x+y)^Z$.

Solution: 64

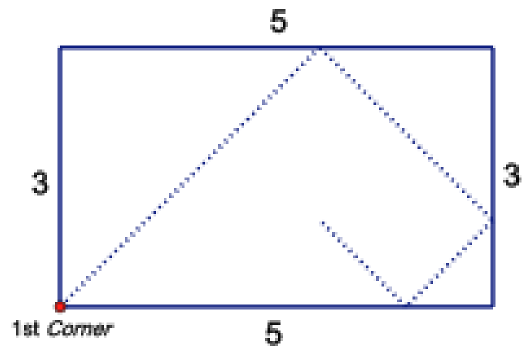
   (d) Let $Z = \sqrt[3]{c}$.

   If $\dfrac{m}{n} = \dfrac{Z}{3}$ and $\dfrac{r}{t} = \dfrac{9}{14}$, find the value of $\dfrac{3mr - nt}{4nt - 7mr}$.

Solution: $-\dfrac{11}{14}$

2. Relay 2: (Is this right? It's either this problem or the problem at the bottom of this file)

(a) A ball is rolled from the corner of a 3mm $\times$ 5mm room and it continually rolls off each wall at a 45-degree angle (see figure below). The ball travels $x\sqrt{2}$ mm before reaching another corner. Find x.



Solution: 15

(b) Let $Z = a - 7$.

If a dealer could get his goods for Z% less while keeping his selling price fixed, his profit (profit= selling price minus cost) would increase by 10%.
The dealers' profit to cost ratio would be $x : 5$. Determine x.

Solution: 4

(c) Let $Z = \frac{3}{b}$.

A man walked a certain distance at a constant rate. If he had gone Z miles per hour faster, he would have walked the distance in $\frac{4}{5}^{th}$ the time. If he had gone Z miles per hour slower, he would have been 2 hours longer on the road. Find the distance in miles he walked.

Solution: 18

(d) Let $Z = -\frac{c}{36}$.

An infinite geometric series has 1^{st} term a and common ration Z with $|Z| < 1$. The sum of this series is S with $S > 0$. The sum of the cubes of the terms of the series is $12S$. Find a.

Solution: 3

3. Relay 3:

(a) Consider the following true statements.

- There exists at least one wizard.
- Every wizard knows exactly three spells.
- Every spell is known by exactly two wizards.
- Every pair of wizards knows exactly one common spell.

Exactly how many wizards must there be?

Solution: 4

(b) Let $Z = a + 6$.

There are Z students in a Defense Against the Dark Arts class. The average score on the last exam was an 84. Neville scored a 50 and Ron scored a 70. Professor Moody wants to remove these scores and find the class average for everyone else. What is the average for everyone else in the class?

Solution: 90

(c) Let $Z = \dfrac{b}{15}$.

Hermione is sending Harry a message using Morse code. In Morse code, symbols are represented by variable length sequences of dashes and dots. For example, A has length 2 and is given by $A = \cdot -$ while 1 has length 5 and is given by $1 = \cdot - - - -$

How many different symbols can be represented by sequences of Z or fewer dots and dashes.

Solution: 126

(d) Let $Z = \dfrac{c}{42}$.

Professor McGonagall buys a box of doughnuts for her transfigurations class. Two students each eat twice as many as a third student. Another Z students eat two each. What is the smallest number of doughnuts the box could have contained if she purchased an odd number and all students ate at least one?

Solution: 11

4. Relay 4:

(a) Find the number of distinct points common to the curve $x^2 + 4y^2 = 1$ and $4x^2 + y^2 = 4$.

Solution: 2

(b) Let $Z = a + 1$.

The length of rectangle $ABCD$ is 5 inches and its width Z inches. Diagonal AC is divided into three equal segments by points E and F. Find the area of $\triangle BEF$ expressed in square inches.

Solution: $\frac{5}{2}$

(c) Let $Z = 6b$.

Given the ratio $3x - 4$ to $y + Z$ is a constant, and $y = 3$ when $x = 2$, then, find x when $y = 30$.

Solution: 3

(d) Let $Z = 2c$.

If the arithmetic mean of two numbers is Z, and their geometric mean is 10, find an equation with the given two numbers as roots.

Solution: $x^2 - 12x + 100$

5. Relay 5:

(a) Elsa and Anna are planning a cruise for the people of Arendelle. The average weight of the 100 people who will be attending is 130 lbs. There are 20 girls with an average weight of 70 lbs,15 boys with an average of 80 lbs, and 65 adults. What is the average weight for all the adults?

> **Solution:** 160

(b) Let $Z = \dfrac{a}{16}$

Remy sent Linguini to the store to buy some apples. He asked for 50 cents worth. Linguini was surprised to see he received 5 more apples than the week before (for the same price!). While leaving the market, he saw a sign stating the price had dropped by Z cents per dozen. What was the new price of apples in cents per dozen?

> **Solution:** 30

(c) Let $Z = b + 100$.

In a group of five minions, the sum of the ages of each group of four of them are $124, 128, 136, 142$ and Z. What is the age of the youngest minion?

> **Solution:** 23

(d) Let $Z = c + 27$.

Rapunzel starts moving north at 4 miles/hour from the palace. Five hours later, Flynn Rider starts moving south 2 miles/hour from a point Z miles due east of the palace. What is the distance (in miles) between Rapunzel and Flynn 5 hours after Flynn starts moving?

> **Solution:** $50\sqrt{2}$

6. Relay 6:

   (a) Four students are to be chosen from a group of ten students, five boys and five girls, to compete in a math competition. How many distinct choices of four students are possible if two boys and two girls must be selected?

   Solution: 100

   (b) Let $Z = \dfrac{a}{10}$.

   If there are Z students, half of whom are male and half female, in how many ways can four be selected to compete in a math competition if at least one student must be female?

   Solution: 205

   (c) Let $Z = \dfrac{b-5}{25}$.

   If there are Z students of whom four are to be chosen to compete in a math competition, how many selections of four are possible if two of the students will only compete if they can both compete?

   Solution: 30

   (d) Let $Z = \dfrac{c}{5}$.

   If there are Z students of whom four are to be chosen to compete in a math competition, how many selections of four are possible if two students refuse to both go to the competition; each of the two will only compete if the other does not.

   Solution: 9

7. Relay 7:

   (a) Two swimmers, at opposite ends of a 90-foot pool, start to swim the length of the pool, one at a rate of 3 feet per second and the other at a rate of 2 feet per second. They swim back and forth for 12 minutes. Allowing no loss of time at the turns, find the number of times they pass each other.

   Solution: 20

   (b) Let $Z = 500a$.

   If you have Z dollars to invest, and you invest $\$4,000$ at 5% interest per year, $\$3,500$ at 4% interest per year, find the rate at which you must invest the remaining amount in order to make $500 per year. Present your answer to the next person without a percent symbol.

   Solution: 6.4

   (c) Let $Z = 50(b - 0.4)$.

   A girls camp is located Z feet from a straight road. On this road a boys camp is located 500 feet from the girls camp. If a dining hall is to be built on the road exactly the same distance from each camp, find the number of feet from the dining hall to each camp.

   Solution: 312.5

   (d) Let $Z = c - 240.5$.

   Two men set out at the same time to walk towards each other from Happytown and Singville, which are Z miles apart. The first man walks at a rate of 4 mph. The second man walks 2 miles the first hour, 2.5 miles the second hour, 3 miles the third hour, and so on in arithmetic progression. After hour many hours will the men meet?

   Solution: 9