KCMC 2022 Relays

1. (a) Determine the smallest positive integer a such that $3780 \cdot a$ is a perfect square.

   (b) Let $Z = \dfrac{a}{5} - 10$

   A jar contains some coins, potentially quarters, dimes, nickels, and pennies, and the average value of these coins is Z cents. When another nickel and three pennies are added to the collection, the average value of each coin becomes 7 cents. How many coins were originally in the jar?

   (c) Let $Z = 4b - 15$.

   A six-sided die, with face numbers 1, 2, 3, 4, 5, 6, is weighted so that the probability of a number being rolled is proportional to that number. What is the probability of rolling a number that is less than Z?

   (d) Let $Z = \dfrac{10}{3c}$.

   Solve for the value of x such that the area of a square with each side length $x + Z$ equals the area of a right triangle with both side lengths equal to $2x$.

2. (a) You roll a fair six-sided die 3 times. What is the probability all three die come up as the same number?

(b) Let $Z = 324a$.

Solve the following system of equations for b.

$$4b + 3y = 7$$
$$-2b - 9y = Z$$

(c) Let $Z = b$.

Let $f(x)$ be the equation of the line perpendicular to the line $y = Zx + 7$ and through the point $(12, -5)$. Find $f(18)$

(d) Let $Z = 5 - c$

The average of three numbers is 13. The second number is four larger than the first number. The average of the second and third number is Z. Let d be the largest of the three numbers.

3. (a) Determine the number of triples of positive integers (x, y, z) such that $1 < x < y < z$ and $x + y + z = 21$.

(b) Let $Z = \dfrac{a + 11}{10}$.

Suppose there are 2 mystery books, 4 biographies, and Z comic books in a pile. In how many ways can all these books be arranged on a shelf if books of the same genre must be adjacent to each other?

(c) Let $Z = \dfrac{b}{72} - 10$.

There are 80 people at a breakfast buffet; 40 of them choose muffins, 36 choose bagels, and 34 choose fruit cups. Of these, 28 of them choose bagels and fruit, 21 choose muffins and fruit, 17 choose bagels and muffins, Z choose all three. How many people do not choose any muffins nor bagels nor fruit cups.

(d) Let $Z = \dfrac{c - 8}{2} - 4$.

Square A has side length z and is contained in a larger square B such that the side length of square B equals the length of the diagonal of square A.

Similarly, square B is contained in a larger square C such that the side length of square C equals the length of the diagonal of square B.

Square C is contained in a circle such that the the diameter of the circle equals the length of the diagonal of square C. Find the area of the circle.
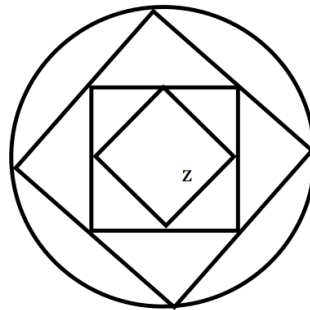
4. (a) How many integers, n for $1 \leq n \leq 100$ are divisible by at least one of 3 and 5?

(b) Let $Z = a - 43$.

Billy-Bob started at home and took a ride on his quad. Unfortunately it ran out of gas and he needed to walk back home. If the quad averaged 15mph, and he walked back at a rate of 3mph, how many miles did he ride if he arrived back home in Z hours?

(Note: Assume he took the same route both on the quad and walking with no detours.)

(c) Let $Z = b - 6$

There are 8 democrats, 10 republicans, and Z independents elected in a state. They need to form a new committee with 4 democrats, 4 republicans, and 2 independents. How many ways can you form the committee?

(d) Let $Z = \dfrac{1}{10} \cdot \left(\dfrac{c}{100} - 2 \right) - 74$

Consider the equations $|x| + x + y = 12$ and $x + |y| - y = Z$. Find $\dfrac{x}{y}$ in reduced terms.

5. (a) Express 17 as the difference of two squares. What is the larger of the two numbers being squared? (i.e. If $17 = a^2 - b^2$, what is a?)

(b) Let $Z = a$.

Suppose there are Z people at a party and each person shakes hands with everyone else at the party exactly once. How many handshakes take place?

(c) Let $Z = 2b + 1$.

Determine the value of n such that $1 + 2^n + 4^n = Z$.

(d) Let $Z = c + 1$.

Calculate the distance between the points of intersection of $y = x^2 + x$ and $y = 3x + Z$.

6. (a) Suppose we group sets of 3 positive integers as follows: Set 1: $\{1, 3, 5\}$, Set 2: $\{7, 9, 11\}$, Set 3: $\{13, 15, 17\}$, etc. What is the sum of the numbers in Set 2022?

(b) Let $Z = \dfrac{b - 387}{12,000}$

Jerry and Duncan are running a 150 meter race. Jerry runs at a constant speed of 5 meters per second, and Duncan runs at a constant speed of Z meters per second. How many more seconds does it take Duncan to finish the race?

(c) Let $Z = b + 10$.

The distance between floors for a building is the distance from the bottom of one floor to the bottom of the next floor. In the Holy Cross dorm, the distance between floors 4 and 6 is Z meters and the distance between adjacent floors is the same throughout the building. If the Holy Cross dorm has 9 floors, what is the distance between floor 1 and floor 9?

(d) Let $Z = \dfrac{c}{4}$

Betty would like to buy some figurines. She can buy 2 figurines for \$4 or 5 figurines for \$8. If she has Z dollars to spend, what is the maximum number of figurines she can buy?