

KCMC 2021 Relays

1. (a) In how many ways can a group of six friends sit around a round table with six seats?

(b) Let $Z = \frac{a}{4}$

Find b after simplifying the following product of consecutive powers of 5:

$$5^1 \cdot 5^2 \cdot 5^3 \cdot 5^4 \cdots 5^Z = 5^b$$

(c) Let $Z = \frac{b-5}{10}$.

Suppose a group of Z students takes a test graded out of 100 points and the average is 72. If the lowest score of the group is removed, the average of the remaining students is a 73. What is the lowest score on the test?

(d) Let $Z = \frac{c+1}{2}$.

Determine the number of possible positive integers values of n for which $\frac{8n+Z}{n-2}$ is an integer.

2. (a) You are in charge of putting away the laundry. There are 25 socks you need to put into a dresser with four drawers. Let x be the number of socks in the drawer with the most socks. What is the difference between the largest possible value of x and the smallest possible value of x ?

(b) Let $Z = \frac{a}{3}$.

In Z years, Stephanie's age will be twice that of Michelle's. Z years ago, Stephanie's age was four times Michelle's. How old will Michelle be Z years from now?

(c) Let $Z = b - 11$.

Sara only completed Z out of 7 problems on a homework assignment. If her professor randomly grades 3 problems, what is the probability Sara completed all 3?

(d) Let $Z = 35c + 997$.

A movie ticket normally costs \$10. During the matinee, a ticket costs 70% of the normal price. If 30% of Monday's tickets sold were for the matinee and the movie theater made $\$Z$, how many total tickets were sold on Monday?

3. (a) Suppose you flip a coin 6 times. What is the probability that you get heads exactly once or exactly twice?

(b) Let $Z = \frac{16}{7}a + \frac{5}{4}$.

Find the value of the sum $Z + Z^2 + Z^3 + Z^4 + Z^5 + Z^6 + Z^7$.

(c) Let $Z = \frac{b-4}{25} - 5$.

There were 189 people at a party. Of these, 36 were eating popcorn, 53 were drinking lemonade, 49 were eating cookies, 27 were eating popcorn and drinking lemonade, 18 were drinking lemonade and eating cookies, 12 were eating popcorn and cookies, and Z were eating popcorn and cookies and drinking lemonade.

How many people at the party were not eating popcorn nor drinking lemonade nor eating cookies?

(d) Let $Z = \frac{c-8}{19} + 2$.

Suppose three people each roll a die. Find the probability that the sum of the three numbers rolled is Z .

4. (a) You have 2021 marbles and 9 buckets. You place a marble in bucket 1, bucket 2, and continue placing until bucket 9. You then start again placing a marble in bucket 1. In what bucket will you place your last marble?

- (b) Let $Z = a - 2$.

You have three containers, b , k and l , that are partially filled with water. If Z quarts from k are poured into b , then the new amount in b will contain as much water as the new amount in k and l combined. If instead from the original, we pour 5 quarts from b into l , then the new amount in l contains as much as the amount in k and the new amount in b contains 2 quarts more than k . How many quarts of water were in the original container b ?

- (c) Let $Z = 4b - 1$.

If Nick earns a 71 on his next quiz, his quiz average will be 83. If he earns a Z , his quiz average will be an 87. How many quizzes did Nick already take?

- (d) Let $Z = c - 1$.

You are given Z balls to throw into Z empty baskets. You randomly select which basket to throw each ball into. After you finish throwing the balls, there is exactly one empty basket. How many ways could you have thrown the balls to obtain that result?

5. (a) Find the largest positive integer n for which $(108)^n$ divides $33! = 33 \cdot 32 \cdot 31 \cdot 30 \cdot 29 \cdots 3 \cdot 2 \cdot 1$.

(b) Let $Z = a$.

If Z people, whose names start with Z distinct letters, randomly line up to get into a movie theater, find the probability that they line up in alphabetical order.

(c) Let $Z = 5 \cdot \left(\frac{1}{10b} - 2 \right)$.

It takes Jake Z minutes to paint a certain room and it takes Emma 40 minutes to paint the same room while working at a constant pace. If Jake and Emma decide to paint this room together at their usual paces, starting at the same time with neither painting any portions that the other has painted, how many minutes will it take them to paint the room together?

(d) Let $Z = \frac{54c}{100}$.

If you randomly choose 2 integers without replacement from the first Z positive integers: $1, 2, 3, \dots, Z$, what is the probability that you choose two integers that differ by exactly 1?

6. (a) 40 students signed up to play intramural basketball. Your school decides each team must contain exactly 9 students and there must be an even number of teams. What is the least number of additional students you need to sign up to make teams?

(b) Let $Z = 3a + 10$.

Let w , x and y be integers such that $2w + 3x = Z$, $3x + y = 41$, and $xy = 60$. Find $w + x + y$.

(c) Let $Z = \frac{2}{5}b$.

Consider the integers $1, 2, \dots, Z$. You randomly select a sequence of 9 integers from this list without repetition. How many ways can you select your sequence so that the numbers are increasing?

(d) Let $Z = 101c$.

Consider the set $\{1, 2, \dots, 2021\}$. How many ways can you create two subsets, such that one subset contains $2Z$ elements?