KCMC 2019 Relays

1. (a) Let x be a real number that satisfies the equation $\sqrt{x+7} + \sqrt[4]{x+7} = 30$. What is x?

   (b) Let $Z = \dfrac{a+2}{20} - 7$.

   Suppose there are Z people at a party and each person wants to shake hands with everyone else exactly once. How many handshakes will take place if they are successful? Your answer must be presented as a single integer.

   (c) Let $Z = \dfrac{b+24}{10}$.

   Suppose a large box weighs Z pounds plus a third of its own weight. How many pounds does the box weigh?

   (d) Let $Z = c$.
   If you and your best friend are among Z people to be seated randomly in a row of Z chairs at a theater, find the probability that you and your best friend will sit next to each other. Your answer must be given as a reduced fraction.

2. (a) You roll two dice. What is the probability you get a sum of 6 or sum of 8? Provide your answer as a reduced fraction.

(b) Let $Z = 108a$.

Solve the following system of equations for b.

$$3x + 4b = \quad Z$$
$$x - 7b = \quad -40$$

(c) Let $Z = b - 2$.

Bertha was asked to add 5 to certain number and then multiply by Z. Instead she added Z and multiplied by 5 to get 100. If she had done the problem correctly, what would the answer have been?

(d) Let $Z = \dfrac{c}{21}$.

Consider set $A = \{1, 2, 3, \ldots 10\}$, where $Z \in A$. How many subsets of A must contain Z and all multiples of Z?

3. (a) Marty tell only lies on Monday, Tuesdays, and Wednesdays but tells only the truth on all other days.

Carla tells only lies on Thursdays, Friday, and Saturdays, but tells only the truth on all other days.

On what day of the week are they both able to say, "I lied yesterday"?

Provide your answer in numerical form according to the following rule:

- Sunday $= 1$
- Monday $= 2$
- Tuesday $= 3$
- Wednesday $= 4$
- Thursday $= 5$
- Friday $= 6$
- Saturday $= 7$

(b) Let $Z = a$.

There are Z pumpkins lined up in a row by weight. The weights (in pounds) of all these pumpkins are consecutive odd numbers. The average weight of all the pumpkins is Z pounds. How many pounds does the heaviest pumpkin weight?

(c) Let $Z = b - 1$.

Sarah is Z years older than Allie. Z years ago Sarah was twice as old as Allie was. How old is Allie now?

(d) Let $Z = c + 7$.

What is the sum of the digits of $10^Z - Z$?

4. (a) Fred, George, and Ron planned on snacking on Chocolate Frogs once they board the Hogwarts express, but one of them ate all the Frogs before they boarded. They say the following:

Fred: George ate all the Chocolate Frogs

George: Ron did not eat the Chocolate Frogs

Ron: Fred is lying

If exactly one of them is telling the truth, who ate all the Chocolate Frogs?
Provide your answer as follows: 1. Fred, 2. George, 3. Ron.

(b) Let $Z = 24a$.

Ten students take a math test and score an average of 76. If five of these students had an average score of Z, what is the average of the other 5 students?

(c) Let $Z = b$.

There are 100 people, of which 70 like T. Swift, 60 like Selena Gomez, and Z like Beyonce. What is the smallest number of people that can like all three?

(d) Let $Z = 39c$.

The pages of a book are numbered consecutively 1 to Z. How many times does a 9 appear in the numbering?

5. (a) An elf has 140 chocolates, each wrapped in either red, green, or silver foil. He distributes all the chocolates among two jars.

In Jar 1 the ratio of red to silver wrapped chocolates is 2 : 3 and the ratio of silver to green is 4 : 5.

In Jar 2 the ratio of red to silver wrapped chocolates is 2 : 3 and the ratio of silver to green is 3 : 5.

How many of the 140 chocolates are wrapped in silver?

(b) Let $Z = \dfrac{a}{9}$.

A cube and a cylinder have the same volume. The radius of the cylinder is equal to the length of one side of the cube, which is Z inches. Find the height of the cylinder in inches, and round your answer to the nearest integer.

(c) Let $Z = b$.

Find the sum of the 956th through 961st terms of the sequence given by
$$2, -7, 8, Z, 5, 11, 2, -7, 8, Z, 5, 11, 2, -7, 8, Z, \ldots$$

(d) Let $Z = 10c - 12$.

There are 300 people at a chocolate party. Of these, Z people are drinking hot chocolate, and the following data was collected about their drinks.

- 80 added marshmallows
- 62 added milk
- 45 added a peppermint stick
- 35 added both marshmallows and milk
- 21 added both milk and a peppermint stick
- 20 added both marshmallows and a peppermint stick
- 9 people added all three: marshmallows and milk and a peppermint stick

How many people drinking hot chocolate did not add any milk nor marshmallows nor a peppermint stick?

6. (a) Consider an 6×6 chessboard. Randomly choose 2 spaces to place opposing queens. What is the probability the queens can attack? Recall: Queens can attack by moving any number of spaces vertically, horizontally, or diagonally in both directions.

   Provide your answer as a reduced fraction.

   (b) Let $Z = 35a + 60$.

   You want to distribute Z puppies among 10 people in such a way that no person has the same number of puppies and each person has at least one puppy. If you are one of the 10 people, what is the largest number of puppies you can take?

   (c) Let $Z = \dfrac{b}{11}$.

   Tina has a total of 10 coins (pennies, nickels, dimes, or quarters) in her pocket for a total of 53 cents. If she randomly selects Z coins, what is the probability they are all distinct monetary values?

   Provide your answer as a reduced fraction.

   (d) Let $Z = 30c + 1$.

   Consider set $A = \{1, 2, 3, \ldots Z\}$. How many non-empty subsets of A have the same amount of even numbers as odd numbers? For example: $\{1, 2, 3, 4\}$ and $\{1, 2\}$ would be such subsets.