

KCMC 2018 Relays

1. (a) If two digits are randomly selected without replacement from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and added together, what is the probability that their sum is 11?

(b) Let $Z = 27a$.

Find the sum of the x and y -coordinates of the midpoint of the line segment with endpoints $(Z, -8)$ and $(-6, 6)$.

(c) Let $Z = -30b$.

A box with a square base and no top is to be made from a square piece of cardboard by cutting a 3×3 in.² square from each corner and folding up the sides. If the box is to hold Z in.³, what is the length (in inches) of one side of the square piece of cardboard to be used to make the box?

(d) Let $Z = \frac{c - 6}{5}$.

Solve the equation $\sqrt{2x} = \sqrt{x + 1} + Z$ for x .

2. (a) Consider the function $f(a) = a^3 - 2a^2 + 4a - 8$. Find the largest positive root.

(b) Let $Z = a + 3$.

A number is pretty if it is divisible by 5 and has only odd digits. A nice number is a pretty number that has Z digits. How many nice numbers are there?

(c) Let $Z = \frac{b}{25} - 23$.

Suzy buys lemons at Z for 50 cents. She resells them at Z for 60 cents. How many lemons does she need to sell to make a profit of \$2.00?

(d) Let $Z = \frac{c}{10}$.

Suppose points $(1, y)$ and $(Z, -2)$ are on line L_1 and points $(2, 8)$ and $(-7, y + 4)$ are on L_2 . If L_1 and L_2 are parallel, what is y ?

3. (a) A passenger train and a freight train leave town at the same time and travel in opposite directions at speeds of 60 mph and 75 mph, respectively. How many hours (expressed as a decimal) will it take the two trains to be 297 miles apart?

(b) Let $Z = \frac{10a - 4}{3}$.

If 3 men and $Z - 3$ women randomly sit in a row of Z seats, what is the probability that the 3 men will be seated in 3 adjacent seats?

(c) Let $Z = 10b + 5$.

From a club with Z members, including John and Sarah, 4 must be chosen to serve on a leadership board. In how many ways can 4 people be chosen if at least one of John and Sarah must be chosen?

(d) Let $Z = \frac{c + 2}{4}$.

Find the value of $\sum_{i=1}^{Z-5} 3^{i-1} - \sum_{i=1}^{Z+1} (-1)^{i-1}$.

4. (a) Dwight has just given Jim a demerit. Jim asks what that means. Dwight tells him that three demerits earns you a citation. Five citations, and you're looking at a violation. Four of those, and you'll receive a verbal warning. Two of those, that will land you in a world of hurt, in the form of a disciplinary review, written up by me, and placed on the desk of my immediate superior. Let a be the number of demerits needed to have a disciplinary review. Find a .

(b) Let $Z = \frac{a}{60}$.

Michael Scott is running a "fun run" to help raise awareness for a cure for rabies. He runs the first 3 miles with a speed of 3 mph and the next three miles with a speed of Z mph. Let b be the time it took him to finish the race in minutes. What is b ?

(c) Let $Z = b + 120$.

Google Earth has captured Dwight and Mose on a see-saw on the Schrute family farm. The see-saw is slightly off center. When Dwight is on the left side and Mose, who weighs 120 pounds, is on the right side, they are perfectly balanced. When Kevin, who weighs Z pounds, sits on the left and Dwight is on the right, they are perfectly balanced. Assuming the see-saw weight is negligible, how much does Dwight weigh?

(d) Let $Z = \frac{c}{18}$.

There are Z people attending a meeting at Dunder-Mifflin. If Michael Scott and Toby sit next to each other, a fight will break out. It is equally likely where each person in the meeting will sit. If they sit at a circular table, what is the probability a fight doesn't break out?

5. (a) Machine A can complete a job in 6 hours when working alone. Machine B can complete the same job in 10 hours, working alone. How many hours will it take both machines, working together the whole time, to complete the same job?

(b) Let $Z = 10a - 1.5$.

Find the largest real solution to $x^4 - 5x^2 - Z = 0$.

(c) Let $Z = 2\sqrt{b}$.

Find the value of the angle (in radians) between the vector $\langle 2, Z \rangle$ and the x -axis.

(d) Let $Z = \frac{12c}{\pi} - 2$.

In how many ways can $6Z$ people be divided into 3 groups, containing Z , $2Z$, and $3Z$ people, respectively?

6. (a) A container contains red and green jelly beans. One jelly bean is drawn at random. If there were two less green jelly beans and two more red jelly beans, the probability of drawing a green jelly bean is $\frac{1}{4}$. If there were two more green jelly beans and two less red jelly beans, the probability of drawing a green jelly bean is $\frac{1}{3}$. How many red jelly beans are in the container?

(b) Let $Z = \frac{a}{17}$.

Consider the system of equations given by

$$\begin{aligned}\frac{7}{b} + \frac{Z}{y} &= 5 \\ \frac{1}{b} + \frac{4}{y} &= -3.\end{aligned}$$

Find the value of b for the solution of the system.

(c) Let $Z = b + 2$.

Jess rolls a fair six-sided die Z times. What is the probability that the resulting product of the Z rolls is a multiple of 3?

(d) Let $Z = 27b - 16$.

Peter, Samir, and Michael Bolton are filling out TCP reports. Peter can fill out 7 reports in 3 minutes, Samir can fill out 5 reports in 4 minutes, and Michael can staple Z reports in 1 minute. A report must be both filled out and stapled (in either order). How long will it take to fill out 105 forms?