

## KCMC 2017 Relays

1. (a) How many integers from 21 to 897 are divisible by 6?

(b) Let  $Z = \frac{a+4}{50}$ .

Define a function  $f$  such that  $f(x) = dx + c$ , where  $d$  and  $c$  are nonzero constants that satisfy  $f(5) = Z - 2$  and  $f(-3) = 25$ . If  $f(b) = 0$ , what is the value of  $b$ ?

(c) Let  $Z = \frac{3b-1}{3}$ .

What is the coefficient of the term containing  $x^3$  in  $(x+2y)^Z$ ?

(d) Let  $Z = \frac{c-6}{2} - 15$ .

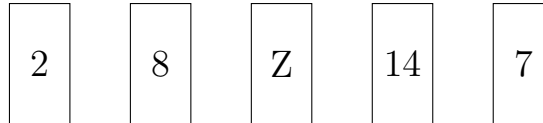
A box is to be made out of a rectangular piece of cardboard, measured in inches, which has a length that is three times longer than its width. To form the box, four  $Z \times Z$  squares of the same size are cut out of the four corners. The remaining flaps are folded up, forming a five-sided box without a top. If the box is to have a volume of  $102 \text{ in}^3$ , how many inches should the width of the starting rectangular piece of cardboard measure?

2. (a) Consider the following true facts:
1. There exists at least one wizard
  2. Every spell is performed by exactly two wizards
  3. Each pair of wizards performs exactly one common spell
  4. Every wizard performs exactly three spells.

Exactly how many wizards must there be?

- (b) Let  $Z = a + 5$ .

Each card below covers a number. The number shown on the card is the sum of the hidden numbers covered by all of the other cards. Find the sum of *all* of the hidden numbers.



- (c) Let  $Z = b - 2$ .

Find the value of  $c$  such that the line  $y = Zx$  intersects the parabola  $y = x^2 + c$  in exactly one point.

- (d) Let  $Z = c - 10$ .

Harry and Ron are two of  $Z$  guys seated at random in a row. What is the probability that exactly two guys are seated between them?

3. (a) Find the smallest positive solution to the equation  $3 \csc^2(2x) = 4$ .

(b) Let  $Z = 8a$ .

If the surface area of a sphere is  $125Z$ , what is its diameter?

(c) Let  $Z = \frac{b}{2}$ .

What is the perimeter of a regular hexagon inscribed in a circle whose radius is  $Z$ ?

(d) Let  $Z = \frac{c}{10}$ .

What is the remainder when  $16Z^2 + 3$  is divided by  $4Z + 2$ ?

4. (a) When I am as old as my mother is now, I will be five times as old as my daughter is now. By then, my daughter will be eight years older than I am now. The sum of my age and my mother's age is 124 years. How much older am I than my daughter?

(b) Let  $Z = a - 23$ .

Compute  $\frac{P(2017, Z)}{C(2017, Z)}$ . Note:  $C(n, k)$  is a combination and can also be written as  ${}_nC_k$  or

$\binom{n}{k}$  and  $P(n, k)$  is a  $k$ -permutation of  $n$  that can also be written as  ${}_nP_k$  or  $P_{n,k}$ .

(c) Let  $Z = \frac{b}{24}$ .

What is the coefficient of  $x^Z y^{8-Z}$  of  $(x - 2y)^8$ ?

(d) Let  $Z = \frac{c}{8} + 64$ .

Find the sum of all of the roots of  $f(x) = 2x^2 - 2x - 12 + (2x + 4)(x - Z)$ .

5. (a) How many distinguishable ordered arrangements of  $ZUZPZU$  exist?

(b) Let  $Z = \frac{a}{10} - 3$ .

How many integers from 1 through  $10^Z$  contain the digit 6 at least once?

(c) Let  $Z = \frac{b+29}{10} - 26$ .

A penny collection contains pennies from each year starting with 1964 up to and including  $1964 + Z - 1$ . If there are one hundred pennies from each of these years and you are to pick some pennies without looking at the dates, how many pennies must you pick to be sure of getting at least five pennies from the same year?

(d) Let  $Z = \frac{c+1}{3}$ .

There are  $Z$  people attending the theater together and they sit in a row with exactly  $Z$  seats. If one is a doctor on call who needs to sit in an aisle seat, in how many ways can they all sit in a row?

6. (a) What is the maximum value of  $x$  such that  $2^x$  divides  $17!$ ?

(b) Let  $Z = a - 3$ .

The average of three numbers is 12. The second number is five smaller than the first number. The average of the second and third number is  $Z$ . What is the largest of the three numbers.

(c) Let  $Z = b - 10$ .

Let  $f(x) = x^2 - Zx - c$ . Find a nonzero value of  $c$  such that  $f(c) = c$ .

(d) Let  $Z = 4c$ .

At the end of a party,  $Z$  handshakes are exchanged. If each guest at the party shook hands with every other guest exactly once, how many guests were at the party?